Synopsis

Ph.D. in Mathematics

Intuitionistic Fuzzy Semi γ^* Generalized Closed Sets

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INTRODUCTION

The notion of fuzzy logic was firstly defined by L. A. Zadeh in 1965. The membership function of an element to a fuzzy set is a value between zero and one. The non-membership function of an element to a fuzzy set is equal to 1 minus the membership degree in fuzzy set theory.

Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a particular object belongs to a set. The notion of fuzzy set theory has caused great interest among both pure and applied mathematics. After introducing fuzzy sets by Zadeh (1965), several kinds of research were conducted on the generalizations of the notion of fuzzy set. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However, in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to one minus the membership degree because there may be some hesitation degree.

Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets have been introduced by Krassimir Atanassov (1983) as an extension of Lotfi Zadeh's notion of fuzzy set, which itself extends the classical notion of a set.

An intuitionistic fuzzy set is useful in decision-making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Recently various applications of intuitionistic fuzzy sets like artificial intelligence, intuitionistic fuzzy expert systems, intuitionistic fuzzy neural networks, intuitionistic fuzzy decision making, intuitionistic fuzzy machine learning, intuitionistic fuzzy semantic representations, etc., have appeared.

Topology provides basic notions to areas of mathematics where there is an essential need for a notion of continuity. Topology focuses on some essential properties of spaces. A set becomes a topological space when each element of the set is given a collection of neighborhoods. Topology is a mathematical subject that studies shapes. The term “Topology” comes from the Greek words “topos" (place) and "ology" (study). Topology is

a part of the geometry that does not include concern distance. General Topology has shown its fruitfulness in both the Pure and Applied directions. It is also called Point-set Topology, which studies fundamentals of neighborhoods, limits, and continuity – basic notions used throughout mathematics.

In 1968, Chang defined the concept of fuzzy topological space and generalized some basic notions of topology such as open set, closed set, continuity and compactness to fuzzy topological spaces. Following this study, Lowen gave another definition of a fuzzy topological space by changing a basic property of topology.

Using the notion of intuitionistic fuzzy set, Coker introduces the notion of intuitionistic fuzzy topological spaces in 1997. After giving the fundamental definitions and the necessary examples he introduced the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy connectedness, intuitionistic fuzzy compactness and obtained several properties.

Hanafy introduced fuzzy γ open sets and fuzzy γ continuity and also he generalized the concept to intuitionistic fuzzy topological spaces in 2009. After the introduction of intuitionistic fuzzy γ continuity by Hanafy. In 2016, Kanimozhi and Jayanthi have introduced intuitionistic fuzzy generalized γ closed sets. Later the concept of fuzzy γ^* generalized closed sets is initiated by Keerthana and Jayanthi in 2017 and in the same year it was generalized to intuitionistic fuzzy topological spaces by Riya and Jayanthi.

This research introduces a new generalization of the closed set called intuitionistic fuzzy semi γ^* generalized closed set in intuitionistic fuzzy topological spaces. Further, the corresponding open sets, theoretical applications, continuous mappings, almost continuous mappings, contra continuous mappings, almost contra continuous mappings, irresolute mappings, completely continuous mappings, perfectly continuous mapping, closed mappings, open mappings, almost closed mappings, contra open mappings, almost contra open mappings, connectedness, and compactness are being defined. Their respective properties, theorems, interrelations between the various closed sets are discussed with necessary counter examples.

Throughout the thesis, the following notations are used.

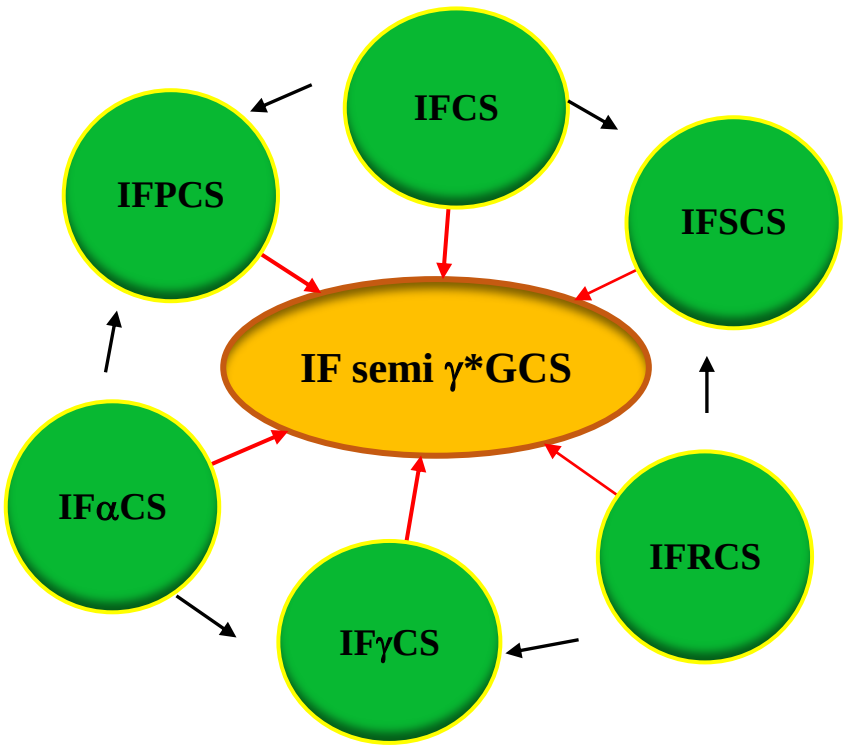
(1) (X, τ), (Y, σ) and (Z, δ) denote the non-empty intuitionistic fuzzy topological spaces on which no separation axioms are mentioned unless explicitly stated.

(2) The closure and interior of a subset A of an intuitionistic fuzzy topological space are denoted by cl(A) and int(A), respectively.

(3) In all the diagrams, A⟶B represents A implies B but not conversely, and A ↮ B represents A and B are independent of each other.

Chapter 1 is divided into six sections that deal with the study of preliminary definitions and results on various already existing closed sets, continuous mappings, closed mappings, connectedness, and compactness in intuitionistic fuzzy topological spaces, which are used to accomplish the work.

Chapter 2 is divided into three sections. Section 2.1 represents the introduction and study of a new class of sets called intuitionistic fuzzy semi γ^* generalized closed sets in intuitionistic fuzzy topological spaces. An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is said to be an **intuitionistic fuzzy semi γ^* generalized closed set** if int(cl(A)) ∩ cl(int(A)) ⊆ U whenever A ⊆ U and U is an intuitionistic fuzzy semi-open set in (X, τ). We have obtained the dependency of intuitionistic fuzzy semi γ^* generalized closed sets with other existing closed sets. In the figure below, we have proved that the converse relations are not generally true with suitable examples.



In section 2.2, we have introduced **intuitionistic fuzzy semi γ^* generalized open sets** in (X, τ), which is the complement of an intuitionistic fuzzy semi γ^* generalized closed sets and investigated their properties.

In section 2.3, **intuitionistic fuzzy semi γ^* generalized $T_{1/2}$ space**, a theoretical application of intuitionistic fuzzy semi γ^* generalized closed set is defined. By using these spaces, we have discussed and proved some results.

Chapter 3 is divided into six sections. In section 3.1, we have introduced a new type of intuitionistic fuzzy continuous mappings called **intuitionistic fuzzy semi γ^* generalized continuous mappings.** A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy semi γ^* generalized continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy semi γ^* generalized closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ). Interrelations with other existing continuous mappings have been obtained and the composition of intuitionistic fuzzy semi γ^* generalized continuous mappings is discussed. Also, we have observed that the converse relations are not true in general. Suitable counter examples are given for the above said relations and their converse.

In section 3.2, a new type of intuitionistic fuzzy contra continuous mappings called **intuitionistic fuzzy contra semi γ^* generalized continuous mapping** is introduced and analyzed. A mapping f: (X, τ) → (Y, σ) is called an **intuitionistic fuzzy contra semi γ^* generalized continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy semi γ^* generalized closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

In section 3.3, intuitionistic fuzzy almost semi γ^* generalized continuous mapping has been introduced, and some of their properties have been studied. A mapping f: (X, τ) → (Y, σ) is called an **intuitionistic fuzzy almost semi γ^* generalized continuous mapping** if f^{-1} (V) is an intuitionistic fuzzy semi γ^* generalized closed set in (X, τ) for every intuitionistic fuzzy regular closed set V of (Y, σ). The interrelations with other existing intuitionistic fuzzy continuous mapping with our newly defined continuous mappings have been established. Further intuitionistic fuzzy almost contra semi γ^* generalized continuous mappings are introduced and their role between in intuitionistic fuzzy topological spaces are discussed.

In section 3.4, we have defined intuitionistic fuzzy semi γ^* generalized irresolute mappings. A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy semi γ^* generalized irresolute mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy semi γ^* generalized closed set in (X, τ) for every intuitionistic fuzzy semi γ^* generalized closed set V of (Y, σ).

In section 3.5, we have introduced completely semi γ^* generalized continuous mappings. We have compared them with other existing intuitionistic fuzzy continuous mappings, intuitionistic fuzzy semi γ^* generalized continuous mappings and intuitionistic fuzzy almost semi γ^* generalized continuous mappings. A mapping f: $(X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic fuzzy completely semi γ^* generalized continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy regular closed set in (X, τ) for every intuitionistic fuzzy semi γ^* generalized closed set V of (Y, σ). Furthermore, we have provided some properties of intuitionistic fuzzy completely semi γ^* generalized continuous mappings and discussed some theorems.

In section 3.6, we have introduced intuitionistic fuzzy perfectly semi γ^* generalized continuous mappings. We have compared them with other existing intuitionistic fuzzy continuous mappings, intuitionistic fuzzy semi γ^* generalized continuous mappings, and intuitionistic fuzzy almost semi γ^* generalized continuous mappings. A mapping f: $(X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic fuzzy perfectly semi γ^* generalized continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy clopen set in (X, τ) for every intuitionistic fuzzy semi γ^* generalized closed set V of (Y, σ). Furthermore, we have provided some properties of intuitionistic fuzzy perfectly semi γ^* generalized continuous mappings and some characterization theorems are discussed.

Chapter 4 is consisting of four sections. In section 4.1, we have introduced intuitionistic fuzzy semi γ^* generalized closed mapping and intuitionistic fuzzy semi γ^* generalized open mapping. A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy semi γ^* generalized closed mapping** if f(V) is an intuitionistic fuzzy γ generalized closed set in Y for every intuitionistic fuzzy closed set V of X. A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy semi γ^* generalized open mapping** if f(V) is an intuitionistic fuzzy semi γ^* generalized open set in Y for every intuitionistic fuzzy open set V of X. Also, the two mappings defined above are equivalent if the mapping is a bijective one.

Section 4.2 deals with the study of intuitionistic fuzzy contra semi γ^* generalized open mappings. A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy contra semi**

γ^* generalized open mapping if f(V) is an intuitionistic fuzzy semi γ^* generalized closed set in Y for every intuitionistic fuzzy open set V of X.

Chapter 5 introduced the notion of **intuitionistic fuzzy semi γ^* generalized connected space** and **intuitionistic fuzzy semi γ^* generalized super connected space**. We have investigated some of their properties and characterized the intuitionistic fuzzy semi γ^* generalized super connected space.

Chapter 6 introduced the notion of intuitionistic fuzzy semi γ^* generalized compact spaces. Theorems, results and examples are given and they have been analyzed elaborately.

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1) **Abinaya, M** and **Jayanthi, D.,** On intuitionistic fuzzy semi γ^* generalized closed sets, Journal of Shanghai Jiaotong University, 2020, 373-387.
2) **Abinaya, M** and **Jayanthi, D.,** On intuitionistic fuzzy semi γ^* generalized continuous mappings, Advances in Fuzzy Mathematics, 2021, 17-26.
3) **Abinaya, M.** and **Jayanthi, D.,** Intuitionistic fuzzy almost semi γ* generalized continuous mappings, Malaya Journal of Matematik, 1, 2021, 278-282.
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6) **Abinaya, M.** and **Jayanthi, D.,** Intuitionistic fuzzy contra semi γ* generalized continuous mappings (to be appeared).
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